Basic Mathematics

The Chain Rule

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The aim of this package is to provide a short self assessment programme for students who want to learn how to use the chain rule of differentiation.

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1. Basic Results

Differentiation is a very powerful mathematical tool. This package reviews the chain rule which enables us to calculate the derivatives of functions of functions, such as $\sin(x^3)$, and also of powers of functions, such as $(5x^2 - 3x)^{17}$. The rule is given without any proof.

It is convenient to list here the derivatives of some simple functions:

y	ax^n	$\sin(ax)$	$\cos(ax)$	e^{ax}	$\ln(x)$
$\dfrac{dy}{dx}$	nax^{n-1}	$a\cos(ax)$	$-a\sin(ax)$	$a\mathrm{e}^{ax}$	$\dfrac{1}{x}$

Also recall the **Sum Rule:**

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

This simply states that the derivative of the sum of two (or more) functions is given by the sum of their derivatives.

It should also be recalled that derivatives commute with constants:

$$\text{i.e., if} \quad y = af(x)\,, \quad \text{then} \quad \frac{dy}{dx} = a\frac{df}{dx}$$

where a is any constant. Here are some chances to practise.

EXERCISE 1. Differentiate the following with respect to x using the above rules (click on the **green** letters for the solutions).

(a) $y = \sqrt{x}$ (b) $y = 4\cos(3x)$

(c) $y = \ln\left(x^3\right)$ (d) $y = 3x^4 - 4x^3$

Quiz Use the properties of powers to find the derivative of $y = \sqrt{w^{\frac{3}{4}}}$ with respect to w.

(a) $\frac{3}{8}w^{-5/8}$ (b) $\frac{3}{4}\sqrt{w^{-1/4}}$ (c) $\frac{3}{8}w^{-1/4}$ (d) $\frac{1}{2}w^{-3/8}$

2. The Chain Rule

The **chain rule** makes it possible to differentiate functions of functions, e.g., if y is a function of u (i.e., $y = f(u)$) and u is a function of x (i.e., $u = g(x)$) then the chain rule states:

$$\text{if} \quad y = f(u)\,, \quad \text{then} \quad \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Example 1 Consider $y = \sin(x^2)\,.$ This can be viewed as $y = \sin(u)$ with $u = x^2.$ Therefore we have

$$\frac{dy}{du} = \cos(u) \qquad \text{and} \qquad \frac{du}{dx} = 2x\,.$$

Thus the chain rule can be used to differentiate y with respect to x as follows:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \cos(u) \times (2x) \\ &= 2x\cos(x^2)\,, \qquad \text{since } u = x^2 \end{aligned}$$

The key to using the chain rule is to choose u appropriately, so that you are able to calculate both of the derivatives $\dfrac{dy}{du}$ and $\dfrac{du}{dx}$. These results can then be substituted into the chain rule to give the desired result $\dfrac{dy}{dx}$.

Quiz To differentiate $y = 3\sqrt{x^3 + 3x}$ with respect to x, what would be a good choice of u?

(a) $3x$ (b) x^3 (c) $x^3 + 3x$ (d) $\sqrt{x^3 + 3x}$

EXERCISE 2. Differentiate the functions y below using the chain rule with the suggested u (click on the **green** letters for the solutions).

(a) $y = \ln(x^7 + x)\,, \quad u = x^7 + x$ (b) $y = \sin(\sqrt{x})\,, \quad u = x^{\frac{1}{2}}$

(c) $y = 3e^{x^3}\,, \quad u = x^3$ (d) $y = \cos(\ln(x))\,, \quad u = \ln(x)$

Exercise 3. Use the chain rule to differentiate the following functions with respect to x (click on the **green** letters for the solutions).

(a)	$y = \sin(x^2)$	(b)	$y = \cos(x^3 - 2x)$
(c)	$y = 2\sqrt{x^2 - 1}$	(d)	$y = 4e^{2x^3} + 2$

Quiz Which of the following is the derivative of $y = 2\sin(3\cos(4t))$ with respect to t?

(a) $6\cos(3\cos(4t))$

(b) $6\cos(12\sin(4t))$

(c) $-24\sin(4t)\cos(3\cos(4t))$

(d) $12\sin(4t)\cos(3\cos(4t))$

3. The Chain Rule for Powers

The **chain rule for powers** tells us how to differentiate a function raised to a power. It states:

$$\boxed{\text{if} \quad y = (f(x))^n\,, \quad \text{then} \quad \frac{dy}{dx} = nf^{'}(x)(f(x))^{n-1}}$$

where $f^{'}(x)$ is the derivative of $f(x)$ with respect to x. This rule is obtained from the chain rule by choosing $u = f(x)$ above.

Proof: If $y = (f(x))^n$, let $u = f(x)$, so $y = u^n$. From the chain rule:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\
&= nu^{n-1}f^{'}(x) = n(f(x))^{n-1} \times f^{'}(x) \\
&= nf^{'}(x)(f(x))^{n-1}
\end{aligned}$$

This special case of the chain rule is often extremely useful.

Example 2 Let $y = (\mathrm{e}^x)^6$. From the chain rule for powers and writing $y = (f(x))^6$ with $f(x) = \mathrm{e}^x$ which also means $f'(x) = \mathrm{e}^x$, we get:

$$\begin{aligned} \frac{dy}{dx} &= 6f'(x)(f(x))^{6-1} \\ &= 6\mathrm{e}^x(\mathrm{e}^x)^5 \\ &= 6\mathrm{e}^x\mathrm{e}^{5x} = 6\mathrm{e}^{6x} \end{aligned}$$

This result can be checked since, from the properties of powers, we can rewrite $y = \mathrm{e}^{6x}$.

Exercise 4. Use the chain rule for powers to differentiate the following functions with respect to x (click on the **green** letters for the solutions).

(a) $y = \sin^5(x)$

(b) $y = (x^3 - 2x)^3$

(c) $y = \sqrt[3]{x^2 + 1}$

(d) $y = 5(\sin(x) + \cos(x))^4$

Finally here are two quizzes:

Quiz Select the derivative $h'(x)$ for the function $h(x) = \sqrt[3]{x^3 + 3x}$ from the choices below:

(a) $\dfrac{-3(x^2+1)}{(x^3+3x)^{\frac{2}{3}}}$ (b) $\dfrac{3x^2+3x}{(x^3+3x)^{\frac{2}{3}}}$ (c) $\dfrac{x^2+1}{(x^3+3x)^{\frac{2}{3}}}$ (d) $\dfrac{3}{(x^3+3x)^{\frac{1}{2}}}$

Quiz Select the derivative $g'(w)$ of $g(w) = \sin^4(w) + \sin(w^4)$ from the choices below:

(a) $4\cos(w)\sin^3(w) + 4w^3\cos(w^4)$ (b) $4\cos^3(w) + 4w^3\cos(w^4)$

(c) $4\sin^3(w)\cos(w) - 4w^3\cos(w^4)$ (d) $-4\sin^3(w) + 4w^3\sin(w^4)$

4. Final Quiz

Begin Quiz Choose the solutions from the options given.

1. What is the derivative with respect to t of $y = 2\mathrm{e}^{-2t^2}$?

(a) $-8t\mathrm{e}^{-2t^2}$ (b) $2\mathrm{e}^{-4t}$ (c) $-4\mathrm{e}^{-2t^2}$ (d) $8t\mathrm{e}^{t^2}$

2. What is the derivative with respect to x of $y = \ln(x^3 + 3)$?

(a) $\dfrac{1}{x^3+3}$ (b) $\dfrac{1}{3x^2}$ (c) $\dfrac{3x^2}{x^3+3}$ (d) $\dfrac{3x^2+3}{x^3+3}$

3. Find the derivative of $y = (w^{33} + 1)^{\frac{1}{33}}$ with respect to w

(a) $(w^{33}+1)^{-\frac{32}{33}}$ (b) $w^{32}(33w^{32})^{-\frac{32}{33}}$

(c) $33w^{32}(w^{33}+1)^{-\frac{32}{33}}$ (d) $w^{32}(w^{33}+1)^{-\frac{32}{33}}$

4. Select below the derivative of $y = 4\sqrt{\sin(ax)}$ with respect to x.

(a) $2a\cos(ax)/\sqrt{(\sin(ax))}$ (b) $4a\cos(ax)/\sqrt{(\sin(ax))}$

(c) $2a/\sqrt{(\sin(ax))}$ (d) $-2a\cos(ax)/\sqrt{(\cos(ax))}$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a) If $y = \sqrt{x}$, then $y = x^{\frac{1}{2}}$ (see the package on **powers**). Its derivative with respect to x is then found with the help of the rule that

$$\frac{d}{dx}\left(ax^n\right) = nax^{n-1}$$

This yields

$$\begin{aligned} \frac{d}{dx}\left(x^{\frac{1}{2}}\right) &= \frac{1}{2} \times x^{\frac{1}{2}-1} \\ &= \frac{1}{2}x^{-\frac{1}{2}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

Click on the **green** square to return

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Exercise 1(b) To differentiate $y = 4\cos(3x)$ with respect to x we take the derivative through the constant 4. We also use the rule

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

In this case with $a = 3$. This gives

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(4\sin(3x)) \\ &= 4\frac{d}{dx}(\sin(3x)) \\ &= 4 \times 3\cos(3x) \\ &= 12\cos(3x) \end{aligned}$$

Click on the **green** square to return

Exercise 1(c) To differentiate $\ln\left(x^3\right)$ with respect to x we first recall from the package on **logarithms** that $\ln(x^n) = n\ln(x)$. This and the rule

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$$

This implies

$$\begin{aligned}\frac{d}{dx}\left(\ln\left(x^3\right)\right) &= \frac{d}{dx}\left(3\ln(x)\right)\\ &= 3\frac{d}{dx}\left(\ln(x)\right)\\ &= 3\times\frac{1}{x}\\ &= \frac{3}{x}\end{aligned}$$

Click on the **green** square to return $\square$

Exercise 1(d) To differentiate $3x^4 - 4x^3$ we use the sum rule and the basic result

$$\frac{d}{dx}x^n = nx^{n-1}$$

This gives

$$\begin{aligned} \frac{d}{dx}\left(3x^4 - 4x^3\right) &= 3 \times 4 \times x^{4-1} - 4 \times 3 \times x^{3-1} \\ &= 12x^3 - 12x^2 \\ &= 12x^2(x-1) \end{aligned}$$

where in the last step we extracted the common factor $12x^2$.

Click on the **green** square to return $\square$

Exercise 2(a) For $y = \ln(x^7 + x)$, **choose** $u = x^7 + x$, i.e., we write

$$y = \ln(u)\,, \qquad \text{where} \quad u = x^7 + x\,.$$

We could then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\left(\ln(u)\right) = \frac{1}{u}\,, \qquad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\left(x^7 + x\right) = 7x^6 + 1\,.$$

Substituting these results into the chain rule yields:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{1}{u} \times (7x^6 + 1) \\ &= \frac{7x^6 + 1}{x^7 + x} \end{aligned}$$

since $u = x^7 + x$.

Click on the **green** square to return

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Exercise 2(b) For $y = \sin(\sqrt{x})$, **choose** $u = \sqrt{x}$, i.e., we write

$$y = \sin(u)\,, \qquad \text{where} \quad u = \sqrt{x} = x^{\frac{1}{2}}\,.$$

We could then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\,(\sin(u)) = \cos(u)\,, \qquad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) = \frac{1}{2}x^{-\frac{1}{2}}\,.$$

These results and the chain rule give:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \cos(u) \times (\frac{1}{2}x^{-\frac{1}{2}}) \\ &= \frac{1}{2\sqrt{x}}\cos(\sqrt{x}) = \frac{\cos(\sqrt{x})}{2\sqrt{x}} \end{aligned}$$

since $u = \sqrt{x}$.

Click on the **green** square to return

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Exercise 2(c) For $y = 3e^{x^3}$, **choose** $u = x^3$, i.e., we write

$$y = 3e^{u}\,, \qquad \text{where} \quad u = x^3\,.$$

We could then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\left(3e^{u}\right) = 3e^{u}\,, \qquad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\left(x^3\right) = 3x^2\,.$$

These results and the chain rule give:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 3e^{u} \times (3x^2) \\ &= 9x^2 e^{u} \\ &= 9x^2 e^{x^3} \end{aligned}$$

since $u = x^3$.

Click on the **green** square to return

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Exercise 2(d) For $y = \cos(\ln(x))$, **choose** $u = \ln(x)$, i.e., we write

$$y = \cos(u)\,, \qquad \text{where} \quad u = \ln(x)\,.$$

We could then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\,(\cos(u)) = -\sin(u)\,, \qquad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\,(\ln(x)) = \frac{1}{x}\,.$$

These results and the chain rule give:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\sin(u) \times (\frac{1}{x}) \\ &= -\frac{\sin(\ln(x))}{x} \end{aligned}$$

since $u = \ln(x)$.

Click on the **green** square to return

Exercise 3(a) For $y = \sin(x^2)$, we define $u = x^2$, so that

$$y = \sin(u)\,, \qquad \text{where} \quad u = x^2\,.$$

We can then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\left(\sin(u)\right) = \cos(u)\,, \qquad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}x^2 = 2x\,.$$

Substituting these results into the chain rule implies:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \cos(u) \times (2x) \\ &= 2x\cos(x^2) \end{aligned}$$

since $u = x^2$.

Click on the **green** square to return

Exercise 3(b) For $y = \cos(x^3 - 2x)$, we define $u = x^3 - 2x$, i.e., we write

$$y = \cos(u)\,, \qquad \text{where} \quad u = x^3 - 2x\,.$$

We can then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\left(\cos(u)\right) = -\sin(u)\,, \quad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\left(x^3 - 2x\right) = 3x^2 - 2\,.$$

Substituting these results into the chain rule implies:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\sin(u) \times (3x^2 - 2) \\ &= -(3x^2 - 2)\sin(x^3 - 2x) \end{aligned}$$

since $u = x^3 - 2x$.

Click on the **green** square to return

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Exercise 3(c) For $y = 2\sqrt{x^2 - 1}$, we define $u = x^2 - 1$, i.e., we write

$$y = 2\sqrt{u} = 2u^{\frac{1}{2}}\,, \qquad \text{where} \quad u = x^2 - 1\,.$$

We can then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\left(2u^{\frac{1}{2}}\right) = 2\frac{1}{2}u^{-\frac{1}{2}} = u^{-\frac{1}{2}}\,, \quad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\left(x^2 - 1\right) = 2x\,.$$

Substituting these results into the chain rule implies:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= u^{-\frac{1}{2}} \times (2x) \\ &= \frac{2x}{\sqrt{x^2 - 1}} \end{aligned}$$

since $u = x^2 - 1$.

Click on the **green** square to return

Exercise 3(d) For $y = 4\mathrm{e}^{2x^3} + 2$, we define $u = 2x^3$, so that

$$y = 4\mathrm{e}^{u} + 2\,, \qquad \text{where} \quad u = 2x^3\,.$$

We can then differentiate y as follows:

$$\frac{dy}{du} = \frac{d}{du}\,(4\mathrm{e}^u + 2) = 4\mathrm{e}^u\,, \quad \text{and} \quad \frac{du}{dx} = \frac{d}{dx}\left(2x^3\right) = 6x^2\,.$$

Note the use of the sum rule to differentiate y above. Putting these results into the chain rule we get:

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 4\mathrm{e}^{u} \times (6x^2) \\ &= 24x^2\mathrm{e}^{2x^3}\end{aligned}$$

since $u = 2x^3$.

Click on the **green** square to return

Exercise 4(a) The function $\sin^5(x)$ is another way of writing $(\sin(x))^5$ Thus to differentiate it we may use the chain rule for powers:

$$\frac{d}{dx}\left((f(x))^n\right) = nf'(x)(f(x))^{n-1}$$

Here we have $f(x) = \sin(x)$ so that:

$$f'(x) = \frac{d}{dx}\left(\sin(x)\right) = \cos(x)$$

so we obtain:

$$\frac{dy}{dx} = 5f^{5-1}(x)\cos(x) = 5\cos(x)\sin^4(x)$$

Click on the **green** square to return

Exercise 4(b) To differentiate $y = (x^3 - 2x)^3$ we may use the chain rule for powers:

$$\frac{d}{dx}\left((f(x))^n\right) = nf'(x)(f(x))^{n-1}$$

Here we have $f(x) = x^3 - 2x$ so that:

$$f'(x) = \frac{d}{dx}\left(x^3 - 2x\right) = 3x^2 - 2$$

so we obtain:

$$\frac{dy}{dx} = (3x^2 - 2) \times 3f^2(x) = 3(3x^2 - 2)(x^3 - 2x)^2$$

Click on the **green** square to return

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Exercise 4(c) To differentiate $y = \sqrt[3]{x^2+1} = (x^2+1)^{\frac{1}{3}}$ we may use the chain rule for powers:

$$\frac{d}{dx}\left((f(x))^n\right) = nf'(x)(f(x))^{n-1}$$

Here we have $f(x) = x^2 + 1$ so that:

$$f'(x) = \frac{d}{dx}\left(x^2+1\right) = 2x$$

so we obtain:

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3} f^{\frac{1}{3}-1}(x) \times 2x \\ &= \frac{1}{3} \times 2x f^{-\frac{2}{3}}(x) \\ &= \frac{2}{3} x(x^2+1)^{-\frac{2}{3}} \end{aligned}$$

Click on the **green** square to return $\square$

Exercise 4(d) To differentiate $y = 5(\sin(x) + \cos(x))^4$ we take the derivative through the factor of 5 at the front and use the chain rule for powers:

$$\frac{d}{dx}\left((f(x))^n\right) = nf'(x)(f(x))^{n-1}$$

Here we have $f(x) = (\sin(x) + \cos(x))$ so that:

$$f'(x) = \frac{d}{dx}\left(\sin(x) + \cos(x)\right) = \cos(x) - \sin(x)$$

so we obtain:

$$\begin{aligned} \frac{dy}{dx} &= 5 \times 4(\cos(x) - \sin(x)) \times f^{4-1}(x) \\ &= 20(\cos(x) - \sin(x))(\sin(x) + \cos(x))^3 \end{aligned}$$

Click on the **green** square to return □

Solutions to Quizzes

Solution to Quiz:

To differentiate $y = \sqrt{w^{\frac{3}{4}}}$ with respect to w, we use the properties of **powers** to rewrite $y = (w^{\frac{3}{4}})^{\frac{1}{2}} = w^{\frac{3}{8}}$. From the basic result

$$\frac{d}{dx}x^n = nx^{n-1}$$

we obtain

$$\begin{aligned} \frac{dy}{dw} &= \frac{d}{dw}\left(w^{\frac{3}{8}}\right) \\ &= \frac{3}{8}w^{\frac{3}{8}-1} \\ &= \frac{3}{8}w^{\frac{3}{8}-\frac{8}{8}} \\ &= \frac{3}{8}w^{-\frac{5}{8}} \end{aligned}$$

End Quiz

Solution to Quiz: Choosing $u = x^3 + 3x$ lets us write

$$y = 3\sqrt{x^3 + 3x} = 3\sqrt{u} = 3u^{\frac{1}{2}}\,.$$

This makes it possible to calculate both $\dfrac{dy}{du} = \frac{3}{2}u^{-\frac{1}{2}}$ and $\dfrac{du}{dx} = 3x^2 + 3.$

These results can be substituted into the chain rule to find

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\
&= \frac{3}{2}u^{-\frac{1}{2}} \times (3x^2 + 3) \\
&= \frac{3}{2}(3x^2 + 3)u^{-\frac{1}{2}} \\
&= \frac{9}{2}(x^2 + 1)(x^3 + 3x)^{-\frac{1}{2}} = \frac{9(x^2 + 1)}{2\sqrt{x^3 + 3x}}
\end{aligned}$$

since $u = x^3 + 3x.$

If we had picked any of the other suggestions for u we would *not* have been able to calculate $\dfrac{dy}{du}$.

End Quiz

Solution to Quiz: For $y = 2\sin(3\cos(4t))$, **choose** $u = 3\cos(4t)$, i.e., we write

$$y = 2\sin(u)\,, \qquad \text{where} \quad u = 3\cos(4t)$$

To differentiate y with respect to t we need:

$$\frac{dy}{du} = \frac{d}{du}\left(2\sin(u)\right) = 2\cos(u)\,, \quad \text{and} \quad \frac{du}{dt} = \frac{d}{dt}\left(3\cos(4t)\right) = -12\sin(4t)$$

These results and the chain rule give:

$$\begin{aligned} \frac{dy}{dt} &= \frac{dy}{du} \times \frac{du}{dt} \\ &= 2\cos(u) \times (-12\sin(4t)) \\ &= -2 \times 12\sin(4t)\cos(u) \\ &= -24\sin(4t)\cos(3\cos(4t)) \end{aligned}$$

since $u = 3\cos(4t)$. End Quiz

Solution to Quiz: The chain rule for powers may be used to differentiate $h(x) = \sqrt[3]{x^3 + 3x} = (x^3 + 3x)^{\frac{1}{3}}$ with respect to x. Writing $h = f(x)^{\frac{1}{3}}$ with $f(x) = x^3 + 3x$ (so that $f'(x) = 3x^2 + 3$) we find that

$$\begin{aligned} h'(x) &= \frac{1}{3} f^{\frac{1}{3}-1}(x) \times (3x^2 + 3) \\ &= \frac{1}{3}(3x^2 + 3) f^{-\frac{2}{3}}(x) \\ &= (x^2 + 1) f^{-\frac{2}{3}}(x) \\ &= \frac{x^2 + 1}{f(x)^{\frac{2}{3}}} \\ &= \frac{x^2 + 1}{(x^3 + 3x)^{\frac{2}{3}}} \end{aligned}$$

where we used $f(x) = x^3 + 3x$ in the last step. End Quiz

Solution to Quiz: To differentiate $g(w) = \sin^4(w) + \sin(w^4)$ with respect to w, we differentiate each of the two terms and add our results.
We use the chain rule for powers to differentiate $\sin^4(w)$ with respect to w. This gives:

$$\frac{d}{dw}\sin^4(w) = 4\sin^3(w)\frac{d}{dw}\sin(w) = 4\cos(w)\sin^3(w)$$

To differentiate $\sin(w^4)$, we write $y = \sin(u)$ where $u = w^4$. From the chain rule (using $\frac{dy}{du} = \cos(u)$ and $\frac{du}{dw} = 4w^3$) we obtain

$$\frac{d}{dw}\sin(w^4) = \frac{d}{du}\sin(u) \times 4w^3 = 4w^3\cos(w^4)$$

Adding these two results gives the final answer:

$$g'(w) = 4\cos(w)\sin^3(w) + 4w^3\cos(w^4)$$

End Quiz